Axioms of Intuition

Kant now turns to working out in detail the schematization of the categories, demonstrating how this supplies us with the principles that govern experience. Prior to doing so he briefly sets out the supreme principles of analytic and synthetic judgment. The supreme principle of analytic judgment is revealed to be the principle of contradiction (A151/B191), a principle that he insists does not require interpretation with regard to time to be built directly into it. In this respect the supreme principle of analytic judgment is distinct from the supreme principle of synthetic judgment. For a synthesis to take place requires something not given in concepts, a "third thing" to unite two divergent notions. Following the doctrine of the schematism this "third thing" is presented as time (A155/B194). To be even more precise: "The synthesis of representations rests on imagination; and their synthetic unity, which is required for judgment, on the unity of apperception" (A155/B194). Kant takes experience itself now to rest upon the synthetic unity of appearances so that even for space and time to have objective validity requires them to be related to the conditions of this synthesis. This provides Kant with the supreme principle of synthetic judgments: "every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience" (A158/B197).

This is given an even fuller explication in the following sentence when Kant succinctly summarizes the results of the investigations of the

Critique up to this point as demonstrating that synthetic *a priori* judgments are made possible by the relationship between the formal conditions of *a priori* intuition, the transcendental synthesis of imagination and transcendental apperception which latter supplies the unity of the former. The combination of these factors supplies the notion of possible empirical knowledge in general. The sentence that concludes this paragraph expresses the view that should result therefore from an understanding of the *Critique* as a whole: "the conditions of the *possibility of experience* in general are likewise conditions of the *possibility of the objects of experience*" (A158/B197).

What Kant now turns to setting out are the fundamental principles on which "the possibility and *a priori* objective validity" of mathematics is grounded (A160/B199). So Kant is not directly here describing principles *of* mathematics but rather the principles that make mathematics itself possible. This requires description of the *a priori* conditions of intuition. These principles are what we will be concerned with this week and next. There are another set of principles that we will turn to after these which Kant terms "dynamical" and those involve an account of the *existence* of an object of possible empirical intuition. Whilst both sets of principles are synthetic *a priori* the former do not require a description of an object, only the conditions of possibility of any object existing under conditions of intuition in general and hence the mathematical principles are logically prior to the dynamical ones and unrestricted in application. Under the heading of mathematical principles Kant is treating of the schematization of the categories of quantity (which we will look at this week) and quality (which we will turn to next time).

The principle of the Axioms of Intuition is stated slightly differently in the two editions with the A-version stating that all appearances are, in their means of being intuited, given as extensive magnitudes. The B-edition version, by contrast, states that all intuitions *are* extensive magnitudes. The difference between the two formulas is not great: the second edition formula states an *a priori* formula clearly (using the criteria of universality) whilst the A-edition appears to restrict the formula though it does not really do so as intuitions are only available to cognition as appearances. The proof of the first edition has an advantage however over that provided in the second edition as Kant in the first edition immediately explains the notion of "extensive magnitude": "I entitle a magnitude extensive when the representation of the parts makes possible, and therefore necessarily precedes the representation of the whole" (A163). Whilst this makes instantly clear that for a magnitude to be *extensive* requires that first the parts of the magnitude have to be given in order for the whole to follow this very explanation has been taken by commentators to be anomalous in the exposition of the *Critique*. Norman Kemp Smith for example states that if this is the account of an "extensive magnitude" then it violates the description in the Transcendental Aesthetic of intuitions where they were described precisely by contrast with concepts as requiring to be given as a whole and parts as only having sense in relation to this whole.

However this objection misrepresents the relationship between the account of the Transcendental Aesthetic and that of the Axioms of Intuition. In the Aesthetic Kant was interested in describing the notion of pure intuition as such, not the conditions for cognition of something in intuition. A pure intuition is contrasted to a pure concept in that in the former a whole has to be given prior to the parts. But the condition for cognition of something by means of intuition is by contrast the means of representation of something as a possible object of intuition as when Kant writes: "I cannot represent to myself a line, however small, without drawing it in thought, that is, generating from a point all its parts one after another" (A162). Successive synthesis is required to cognize something under conditions of intuition, this notion of successive synthesis being in fact the determination of time by means of the synthesis of imagination. When Kant writes in the second edition version of the proof that "as intuitions in space or time, they must be represented through the same synthesis whereby space and time in general are determined" (B203) this is as much as to say that the means whereby space and time are presented are the same means that enable the objects of intuition to be presented. In other terms, without the understanding of homogeneous units as the basis of measurement we cannot be given objects at all so this understanding is the basis of the possibility of intuitions being such that they can yield objects.

Another key point of contrast between the treatment of intuition in the Aesthetic and that in the Axioms is that the Aesthetic describes wholes in general whilst the Axioms are concerned with *determinate* wholes, wholes of certain sorts. A determinate whole is a description of a certain area and such descriptions are only possible given that the intuitive comprehension of space in general is already given. Even the presentation of time requires, as we have noted a number of times already, the representation of space. When something is given in empirical intuition it appears to us by means of a successive advance of parts, a combination and Kant now states that this successive advance is what is at work in the synthesis of imagination, a synthesis that in *generating* figures makes geometry possible (B204). Not only is this the case but for geometrical principles to describe experience requires their connection to the categories as in the statement that a straight line is the shortest distance between two points (as here the reference to quantity is brought in with "shortness", a category, not an intuition).

The pure concepts are now revealed then to be nothing other than concepts of intuition in general. The pure figures of geometry are pure in the sense that they do not arise from sensation or any specific element of space. The reason why they are able to describe experience for us however is that they are generated in the same manner that experience itself is, namely, by means of successive synthesis. If Kant demonstrates that geometry is based on transcendental principles in this way however it is only by means of showing that geometry requires reference to axioms, namely synthetic *a priori* principles that are immediately certain (A733/B761). Kant indicates that whilst such axioms are the basis of geometry and sets these out in Euclidean style (B204: statement of the axioms) he proceeds to deny that the statements of arithmetic are similar in this respect to those of geometry. Kant claims that the statements of arithmetic are not based on axioms but merely on what he terms “numerical formulas”.

This argument is presented in the *Critique* as grounded on the fact that whilst there is only one general formula for how a triangle can be given the means by which it can be presented (in terms of different triangles of different sorts and different heights) is perfectly general whilst a numerical formulae is only given in one way and involves therefore a relation between singular quantities (A64/B205). Another argument indicated in the *Critique* to provide a reason for thinking of geometry and arithmetic as different here is that geometry in being founded on axioms is based on the schema of outer intuition in relying on an account of figures. By contrast no types of figures are necessary for arithmetic, merely numerals. It would seem then that it is not necessary that arithmetic in fact have any “objects” of its own and so we may wonder whether it has any connection to intuition at all. Kant argues that it does as the outcome of an arithmetical sum is not merely contained in its elements so the outcome must be produced by a third thing. This third thing is, he suggests, time so that the “objects” of arithmetic would, strictly understood, be moments.

So what the argument of the Axioms suggests is that space is required for geometry to be possible and that geometry can determine the nature of experience given its relation to the pure intuition of space. However arithmetic, whilst dependent on time to have any "objects" is only determinate in a singular, not a general way. What Kant does think he has done however with his account of the Axioms is to give not only a transcendental basis for the principles on which mathematics must in general rely but also shown a reason for thinking that mathematics does connect to experience via his schematization of the category of quantity.

<u>Axioms of Intuition Seminar</u>

A) Paul Guyer: Raises two problems with the argument of the Axioms. Firstly, we would have expected, given that Kant is here developing his schematism, a serious treatment of time but we mainly get an account of space. Secondly, Kant schematizes quantity in general when we might have expected a treatment of each of the elements of quantity (unity, plurality and totality). However his first point is problematic as the treatment of the schema of sensible concepts in the chapter on schematism involved space and how could it fail to? To this Guyer could reply that we are expecting here is something different, namely a transcendental schema of quantity, not a schema of sensible concepts. However perhaps what the transcendental schema does is show the possibility of the sensible one? In the schematism chapter Kant stated that the schema of quantity would treat the time-series showing the generation of time itself. Hence space gives the condition of time as time can only be represented as a set of moments by means of points. Specific categories: extensive quantity is described as a whole, i.e. plurality as unity. Has Kant shown the connection of pure and empirical intuition to justify geometry? Guyer suggests not. This really rests on the point that we now have non-Euclidean geometries.

B) What is the importance of non-Euclidean geometries? Surely it is not that they show other measurements of space to be conceivable? Kant would never have denied that! He even asserts that there is no logical impossibility in denying Euclidean geometry, a point that follows from his view of it as a body of synthetic *a priori* truths. A different objection would be that in modern geometry we have a distinction between pure and applied geometry with the former being a purely logical theory that does not describe space whilst the latter does describe space but only its contingent properties. The notion of "applied" geometry can be based on statements about straight lines for example but such statements would here be analysed into certain types of pre-given matter that determined straightness. However if an empirical correlate is taken to measure straightness and fails to this does not demonstrate Kant was wrong. Further: argument from incongruent counterparts suggests that experience does include central differences that are intuitive and not merely conceptual. Modern geometry cannot account for this and is hence not transcendentally based.